Quantum Mechanics: FundamentalsAdvanced Quantum MechanicsMathematical Concepts of Quantum Mechanics

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Book reviews

Quantum Mechanics: Fundamentals
Kurt Gottfried and Tung-Mow Yan
2003 Heidelberg: Springer-Verlag
620pp £61.50 (hardback) ISBN 0-387-95576-3

Advanced Quantum Mechanics
Franz Schwabl
2003 Heidelberg: Springer-Verlag
405pp £42.50 (hardback) ISBN 3-540-40152-0

Mathematical Concepts of Quantum Mechanics
Stephen Gustafson and Israel Sigal
2003 Heidelberg: Springer-Verlag
249pp £30.50 (paperback) ISBN 3-540-44160-3

This review is of three books, all published by Springer, all on quantum theory at a level above introductory, but very different in content, style and intended audience.

That of Gottfried and Yan is of exceptional interest, historical and otherwise. It is a second edition of Gottfried's well-known book published by Benjamin in 1966. This was written as a text for a graduate quantum mechanics course, and has become one of the most used and respected accounts of quantum theory, at a level mathematically respectable but not rigorous.

Quantum mechanics was already solidly established by 1966, but this second edition gives an indication of progress made and changes in perspective over the last thirty-five years, and also recognises the very substantial increase in knowledge of quantum theory obtained at the undergraduate level.

Topics absent from the first edition but included in the second include the Feynman path integral, seen in 1966 as an imaginative but not very useful formulation of quantum theory. Feynman methods were given only a cursory mention by Gottfried. Their practical importance has now been fully recognised, and a substantial account of them is provided in the new book. Other new topics include semiclassical quantum mechanics, motion in a magnetic field, the S matrix and inelastic collisions, radiation and scattering of light, identical particle systems and the Dirac equation.

A topic that was all but totally neglected in 1966, but which has flourished increasingly since, is that of the foundations of quantum theory. John Bell's work of the mid-1960s has led to genuine theoretical and experimental achievement, which has facilitated the development of quantum optics and quantum information theory.

Gottfried's 1966 book played a modest part in this development. When Bell became increasingly irritated with the standard theoretical approach to quantum measurement, Viki Weisskopf repeatedly directed him to Gottfried's book. Gottfried had devoted a chapter of his book to these matters, titled 'The Measurement Process and the Statistical Interpretation of Quantum Mechanics'. Gottfried considered the von Neumann or Dirac 'collapse of state-vector' (or 'reduction postulate' or 'projection postulate') was unsatisfactory, as he argued that it led inevitably to the requirement to include 'consciousness' in the theory.

He replaced this by a more mathematically and conceptually sophisticated treatment in which, following measurement, the density matrix of the correlated measured and measuring systems, ρ, is replaced by $\hat{\rho}$, in which the interference terms from ρ have been removed. ρ represents a pure state, and $\hat{\rho}$ a mixture, but Gottfried argued that they are 'indistinguishable', and that we may make our replacement, 'safe in the knowledge that the error will never be found'. Now our combined state is represented as a mixture, it is intuitive, Gottfried argued, to interpret it in a probabilistic way, $|c_m|^2$ being the probability of obtaining the mth measurement result.

Bell liked Gottfried's treatment little more than the cruder 'collapse' idea of von Neumann, and when, shortly before Bell's death, his polemical article 'Against measurement' was published in the August 1990 issue of *Physics World* (pages 33-40), his targets included, not only Landau and Lifshitz's classic Quantum Mechanics, pilloried for its advocacy of old-fashioned collapse, and a paper by van Kampen in *Physica*, but also Gottfried's approach. Bell regarded his replacement of ρ by $\hat{\rho}$ as a 'butchering' of the density matrix, and considered, in any case, that even the butchered density matrix should represent co-existence of different terms, not a set of probabilities.

Gottfried has replied to Bell (*Physics World*, October 1991, pages 34-40; *Nature* 405, 533-36 (2000)). He has also become a major commentator on Bell's work, for example editing the section on quantum foundations in the World Scientific edition of Bell's collected works. Thus it is exceedingly interesting to discover how he has responded to Bell's criticisms in the new edition of the book.

To commence with general discussion of the new book, the authors recognise that the graduate student of today almost certainly has substantial experience of wave mechanics, and is probably familiar with the Dirac formalism. The 1966 edition had what seems, at least in retrospect, a relatively soft opening covering the basic ideas of wave mechanics and a substantial number of applications; it did not reach the Dirac formalism in the first two hundred pages, though it then moved on to tackle rather advanced topics, including a very substantial section on symmetries, which tackled a range of sophisticated issues.

The new edition has been almost entirely rewritten; even at the level of basic text, it is difficult to trace sentences or paragraphs that have moved unscathed from one edition to the next. As well as the new topics, many of the old ones are discussed in much greater depth, and the general organisation is entirely different. As compared with the steady rise in level of the 1966 edition, the level of this book is fairly consistent throughout, and from the perspective of a beginning graduate student, I would estimate, a little tough.

A brief introductory chapter gives a useful, though not particularly straightforward, discussion of complementarity, uncertainty and superposition, and concludes with an informative though very short summary of the discovery of quantum mechanics, together with a few nice photographs of some of its founders.

There follow two substantial chapters which are preparation for the later study of actual systems. The first, called 'The Formal Framework' is a fairly comprehensive survey of the methods of quantum theory—Hilbert space, Dirac notation, mixtures, the density matrix, entanglement, canonical quantization, equations of motion, symmetries, conservation laws, propagators, Green's functions, semiclassical quantum mechanics. The level of mathematical rigour is stated as 'typical of the bulk of theoretical physics literature—slovenly'; those unhappy with this are directed to the well-known books of Jordan and Thirring. The next chapter—'Basic Tools'—explains a set of topics which students will need to use when studying particular systems—angular momentum and its addition, free particles, the two-body system, and the standard approximation techniques.

There follow chapters on low-dimensional systems—harmonic oscillator, Aharanov–Bohm effect, one-dimensional scattering, WKB and so on; hydrogenic atoms—the Kepler problem, fine and hyperfine structure, Zeeman and Stark effects; and on two-electron atoms—spin and statistics. As in the first edition, there is a substantial treatment of symmetries, including time reversal, Galileo transformations, the rotation group, the Wigner-Eckart theorem and the Berry phase. There are two long chapters on scattering—elastic and inelastic respectively, including an account of the S matrix.

The treatment of electrodynamics is much extended and modernised compared to that in the first edition. There are discussions of the quantization of the free field, causality and uncertainty in electrodynamics, vacuum fluctuations including the Casimir effect and the Lamb shift, and radiative transitions. There is a treatment of quantum optics, but this a only a brief introduction to a rapidly expanding subject, designed to facilitate understanding of the experiments on Bell's inequalities discussed in the later chapter on interpretation. Other topics are the photoeffect in hydrogen, scattering of photons, resonant scattering and spontaneous decay.

Identical particles are discussed, with a treatment of second quantization and an introduction to Bose–Einstein condensation, and the last chapter is a brief introduction to relativistic quantum mechanics, including the Dirac equation, the electromagnetic interaction of a Dirac particle, the scattering of ultra-relativistic electrons and a treatment of bound states in a Coulomb field.

Gottfried and Yan's response both to the growing interest in work on foundational matters in general, and to the specific criticism of Bell on the previous edition is included in the chapter entitled 'Interpretation'. This chapter appears to be something of a hybrid. The first four sections broadly discuss hidden variables. An account of the Einstein–Podolsky–Rosen approach is followed by a general study of hidden variables, including a discussion of what the authors call the Bell–Kochen–Specker theorem. Bell's theorem is analysed in some detail; also included are the Clauser–Horne inequality and the experimental test of the Bell inequality by Aspect.

There is an interesting discussion of locality. Granted that both quantum mechanics and experiment (the latter admittedly with a remaining loophole) are in conflict with what the authors call a classical conception of locality as embodied in the Bell inequality, they ask whether quantum mechanics is actually non-local if one uses a definition of locality entailing no ingredients unknown to quantum mechanics. Their answer is that it is a matter of taste. In the statistical distribution of measurement outcomes on separate systems in entangled states, there is no hint of non-locality and no question of superluminal signalling. But quantum mechanics displays perfect correlations between distant outcomes, even though Bell's theorem demonstrates that pre-existing values cannot be assumed.

The second part of this chapter is a discussion of the measurement procedure similar to that in the first edition. The authors aim to show how measurement results are obtained and displayed, and how the appropriate probabilities are determined. The expression of this intention, however, is accompanied by the statement that they are not attempting to derive the statistical interpretation of quantum mechanics, which is assumed, but to examine whether it gives a consistent account of measurement.

The conclusion is that after a measurement, interference terms are 'effectively' absent; the set of 'one-to-one correlations between states of the apparatus and the object' has the same form as that of everyday statistics and is thus a probability distribution. This probability distribution refers to potentialities, only one of which is actually realized in any one trial. Opinions may differ on whether their treatment is any less vulnerable to criticisms such as those of Bell.

To sum up, Gottfried and Yan's book contains a vast amount of knowledge and understanding. As well as explaining the way in which quantum theory works, it attempts to illuminate fundamental aspects of the theory. A typical example is the 'fable' elaborated in Gottfried's article in *Nature* cited above, that if Newton were shown Maxwell's equations and the Lorentz force law, he could deduce the meaning of E and B, but if Maxwell were shown Schrödinger's equation, he could not deduce the meaning of Ψ .

For use with a well-constructed course (and, of course, this is the avowed purpose of the book; a useful range of problems is provided for each chapter), or for the relative expert getting to grips with particular aspects of the subject or aiming for a deeper understanding, the book is certainly ideal. It might be suggested, though, that, even compared to the first edition, the isolated learner might find the wide range of topics, and the very large number of mathematical and conceptual techniques, introduced in necessarily limited space, somewhat overwhelming.

The second book under consideration, that of Schwabl, contains 'Advanced' elements of quantum theory; it is designed for a course following on from one for which Gottfried and Yan, or Schwabl's own 'Quantum Mechanics' might be recommended. It is the second edition in English, and is a translation of the third German edition.

It has a restricted range of general topics, and consists of three parts entitled 'Nonrelativistic Many-Particle Systems', 'Relativistic Wave Equations', and 'Relativistic Fields'. Thus it studies in some depth areas of physics which are either dealt with in an introductory fashion, or not reached at all, by Gottfried and Yan. Despite its more advanced level, this book may actually be the more accessible to an isolated learner, because the various aspects are developed in an unhurried fashion; the author remarks that 'the inclusion of all mathematical steps and full presentation of intermediate calculations ensures ease of understanding'. Many useful student problems are included. The presentation is said to be rigorous, but again this is a book for the physicist rather than the mathematician.

The treatment of many-particle systems begins with a rather general introduction to second quantization, and then applies this formalism to spin-1/2 fermions and bosons. The study of fermions includes consideration of the Fermi sphere, the electron gas, and the Hartree–Fock equations for atoms; that of bosons includes Bose–Einstein condensation, Bogoliubov theory of the weakly interacting Bose gas, and a brief account of superfluidity. The last section of this part of the book investigates in detail the dynamics of many-particle systems on a microscopic quantum-mechanical basis using, in particular, the dynamical correlation functions.

In the second part which considers relativistic wave equations, the Klein–Gordon and Dirac equations are derived, and the Lorentz covariance of the Dirac equation is established. The role of angular momentum in relativistic quantum mechanics is explained, as a preliminary to the study of the energy levels in a Coulomb potential using both the Klein–Gordon and Dirac equations, the latter being solved exactly for the hydrogen atom. For larger atoms, the Foldy–Wouthuysen transformation is explained, and also relativistic corrections and the Lamb shift.

There is an interesting chapter on the physical interpretation of the Dirac equation, including such topics as the negative energy solutions, the Zitterbewegung and the Klein paradox. The last chapter in this part of the book is an extensive treatment of the symmetries and other properties of the Dirac equation, including the behaviour under rotation, translation, reflection, charge conjugation and time reversal. Helicity is explained, and the behaviour of zero-mass fermions is discussed; even though it now seems certain that neutrinos do not have zero-mass, this treatment provides a good approximation to their behaviour if they have high enough momenta.

The last section on relativistic fields contains chapters on the quantization of relativistic fields, the free Klein–Gordon and Dirac fields, quantization of the radiation field, interacting fields and quantum electrodynamics, including the S matrix, Wick's theorem and Feynman diagrams.

Schwabl's book would be excellent for those requiring a detailed presentation of the topics it includes, at a level of rigour appropriate to the physicist. It includes a substantial number of interesting problems.

The third book under consideration, that by Gustafson and Sigal is very different from the others. In academic level, at least the initial sections may actually be slightly lower; the book covers a one-term course taken by senior undergraduates or junior graduate students in mathematics or physics, and the initial chapters are on basic topics, such as the physical background, basic dynamics, observables and the uncertainty principle.

However the level of mathematical sophistication is far higher than in the other books. While the mathematical prerequisites are modest—real and complex analysis, elementary differential equations and preferably Lebesgue integration, a third of the book is made up of what are called mathematical supplements—on operator adjoints, the Fourier transform, tensor products, the trace and trace class operators, the Trotter product formula, operator determinants, the calculus of variations (a substantial treatment in a full chapter), spectral projections, and the projecting-out procedure.

On the basis of these supplements, the level of mathematical sophistication and difficulty is increased substantially in the middle section of the book, where the topics considered are many-particle systems, density matrices, positive temperatures, the Feynman path integral, and quasi-classical analysis, and there is a final substantial step for the concluding chapters on resonances, an introduction to quantum field theory, and quantum electrodynamics of non-relativistic particles. A supplementary chapter contains an interesting approach to the remormalization group due to Bach, Fröhlich and Sigal himself.

This book is well-written, and the topics discussed have been well thought-out. It would provide a useful approach to quantum theory for the mathematician, and would also provide access for the physicist to some mathematically advanced methods and topics, but the physicist would definitely have to be prepared to work hard at the mathematics required.

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